Integrales por Partes

$$\mu \cdot \vartheta - \int \vartheta \, d\mu$$

① $\int x \cdot \cos(x)\, dx$ → Tenemos dos cosas multiplicando y no se puede hacer de la forma "CASI-INMEDIATA".

μ → Derivar

dv → integrar

A - L - P - E - S

A → ARCoSen, ARCoCoSen, ARCotg

L → ln

P → Polinomio

E → e^x

S → { Sen x, Cos x }

$$\boxed{\begin{aligned} \mu &= x \xrightarrow{D} d\mu = 1\,dx \\ dv &= \cos(x)\,dx \xrightarrow{I} v = \operatorname{sen} x \end{aligned}}$$

$$= x \cdot \operatorname{sen}(x) - \int \operatorname{sen} x \, dx = x \cdot \operatorname{sen}(x) - (-\cos(x))$$

$$= x \cdot \operatorname{sen}(x) + \cos(x) + C$$

② $\int x^2 \cdot \operatorname{sen}(x)\,dx$

$\mu = x^2 \xrightarrow{d} d\mu = 2x\,dx$

$d\vartheta = \operatorname{sen}(x)\,dx \xrightarrow{I} \vartheta = -\cos(x)$

$$= x^2 \cdot (-\cos(x)) - \int -\cos(x) \cdot 2x\,dx$$

$$= -x^2 \cdot \cos(x) + 2 \int \cos(x) \cdot x\,dx$$

$\mu = x \xrightarrow{D} d\mu = 1\,dx$

$dv = \cos(x)\,dx \xrightarrow{I} v = \operatorname{sen} x$

Volvemos a hacer otra vez el método de "Por Partes"

$$= -x^2 \cdot \cos(x) + 2 \cdot \left[x \cdot \operatorname{sen}(x) - \int \operatorname{sen}(x) \cdot 1\,dx \right]$$

$$= -x^2 \cdot \cos(x) + 2 \cdot \left[x \cdot \operatorname{sen}(x) - (-\cos(x)) \right]$$

$$= -x^2 \cdot \cos(x) + 2 \cdot \left[x \cdot \operatorname{sen}(x) + \cos(x) \right] + C$$

③ $\int x^2 \cdot \ln(x)\, dx$

$$\boxed{\begin{aligned} u &= \ln(x) \xrightarrow{d} du = \frac{1}{x}\,dx \\ dv &= x^2\,dx \xrightarrow{I} v = \frac{x^3}{3} \end{aligned}}$$

$$= \ln(x) \cdot \frac{x^3}{3} - \int \frac{\cancel{x^3}^{\,2}}{3} \cdot \frac{1}{\cancel{x}}\, dx$$

$$= \ln(x) \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \int x^2\, dx = \ln(x) \cdot \frac{x^3}{3} - \frac{1}{3}\, \frac{x^3}{3}$$

$$= \frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C$$

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(4) $\int e^x \cdot \operatorname{sen}(x)\, dx$

$$\boxed{\begin{array}{l} \mu = e^x \xrightarrow{d} d\mu = e^x dx \\ dv = \operatorname{sen}(x) dx \xrightarrow{I} v = -\cos(x) \end{array}}$$

$$= e^x \cdot (-\cos(x)) - \int (-\cos(x)) \cdot e^x \, dx$$

$$\boxed{\begin{array}{l} \mu = e^x \xrightarrow{d} d\mu = e^x dx \\ dv = \cos(x) dx \xrightarrow{I} v = \operatorname{sen}(x) \end{array}}$$

$$= -e^x \cdot \cos(x) + \int \cos(x) \cdot e^x$$

$$= -e^x \cdot \cos(x) + \left[e^x \cdot \operatorname{sen}(x) - \int \operatorname{sen}(x) \cdot e^x \, dx \right]$$

→ Es la integral original

↓

Se trata de una Integral CÍCLICA

$$\boxed{I = \int \operatorname{sen}(x) \cdot e^x \, dx}$$

$$I = -e^x \cdot \cos(x) + e^x \cdot \operatorname{sen}(x) - I$$

$$2I = -e^x \cdot \cos(x) + e^x \cdot \operatorname{sen}(x)$$

$$\boxed{I = \frac{e^x \cdot (-\cos(x) + \operatorname{sen}(x))}{2} + C}$$

5. $\int e^x \cdot x \, dx$

$$\boxed{\begin{array}{l} u = x \xrightarrow{d} du = 1\,dx \\ dv = e^x \xrightarrow{I} v = e^x \end{array}}$$

$$x \cdot e^x - \int e^x \cdot 1 \, dx = x \cdot e^x - e^x = e^x \cdot (x-1) + C$$

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⑥ $$\int \ln(x)\cdot 1 \, dx$$

¡TRUCO! → Para convertirla en una por partes

$$\boxed{\begin{array}{l} u = \ln x \xrightarrow{d} du = \frac{1}{x}dx \\ dv \to 1\,dx \xrightarrow{\int} v = x \end{array}}$$

$$= \ln(x)\cdot x - \int \cancel{x}\cdot \frac{1}{\cancel{x}}\, dx$$

$$= \ln(x)\cdot x - \int 1\, dx$$

$$= \ln(x)\cdot x - x$$

F.C

$$= x\cdot\left(\ln(x) - 1\right) + c$$

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